

Worksheet #3 - Solution

$$\textcircled{1} f'(x) = 6x^2 - 10x$$

$$\begin{aligned} \textcircled{2} g'(x) &= (2x - 2 + e^x) \left(\frac{1}{x} - e^x \right) + (x^2 - 2x + e^x) \left(-\frac{1}{x^2} - e^x \right) \\ &= (2 - 2xe^x - \frac{2}{x} + 2e^x + \frac{e^x}{x} - e^{2x}) + (-1 - x^2 + \frac{2}{x} + 2xe^x - \frac{e^x}{x^2} - e^{2x}) \\ &= 1 - x^2 - e^x \left(-\frac{1}{x^2} + \frac{1}{x} + 2 - 2e^x \right) \end{aligned}$$

$\textcircled{3}$ $V'(x) = 4\pi x^2$. They correspond to the volume and surface area of a sphere of radius x .

$$\textcircled{4} h'(x) = \frac{2x(x-1) - (x^2-1) \cdot 1}{(x-1)^2} = \frac{x^2 - 2x + 1}{x^2 - 2x + 1} = 1$$

$h'(x)$ has domain $\{x \in \mathbb{R} / x \neq 1\}$

$$\textcircled{5} M'(x) = \frac{(1 + \ln b \cdot b^x)(x^e + e^x) - (x + b^x)(e x^{e-1} + e^x)}{(x^e + e^x)^2}$$

$$= \frac{x^e + \ln b \cdot b^x x^e + e^x + \ln b \cdot b^x e^x - e x^e - x e^x - e b^x x^{e-1} - b^x e^x}{(x^e + e^x)^2}$$

$$= \frac{x^e(1 - e + \ln b \cdot b^x - \frac{e \cdot b^x}{x}) + e^x(1 - x + \ln b \cdot b^x - b^x)}{(x^e + e^x)^2}$$

$$\textcircled{6} \text{ Since } |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases} \Rightarrow |x|^1 = \begin{cases} 1 & \text{if } x > 0 \\ -1 & \text{if } x < 0 \end{cases}$$

$\lceil x \rceil$ (or $\lfloor x \rfloor$) is constant equal to k for $k \leq x < k+1$ (k integer)

$\Rightarrow \lfloor x \rfloor' = 0$, with domain $\{x \in \mathbb{R} / x \text{ is not at integer}\}$

($\lfloor x \rfloor$ is not continuous at the integers)

$$\textcircled{7} \sin(x^2) \quad \text{VS} \quad \sin^2(x)$$

$$\cos(x^2) \cdot 2x$$

$$2\sin(x) \cdot \cos(x)$$